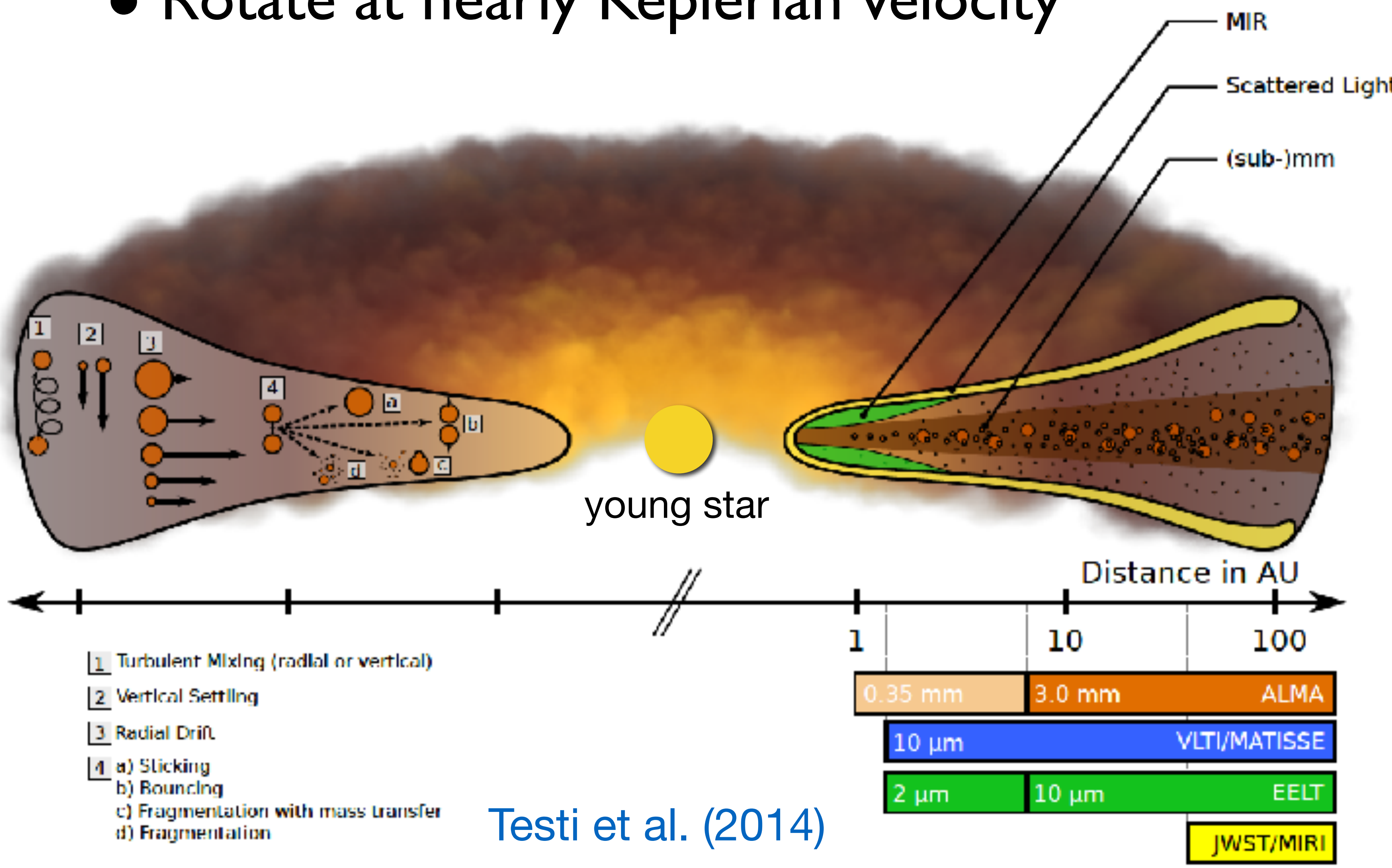
Advanced Earth and Space Sciences B
2019

2. Protoplanetary Disks

Protoplanetary Disk (原始惑星系円盤)

- Disks around young stars (of age < 10 Myr)
- Consist of gas (~99wt%) and dust (~1wt%)
- Radial extent ~ 100 au
- Rotate at nearly Keplerian velocity



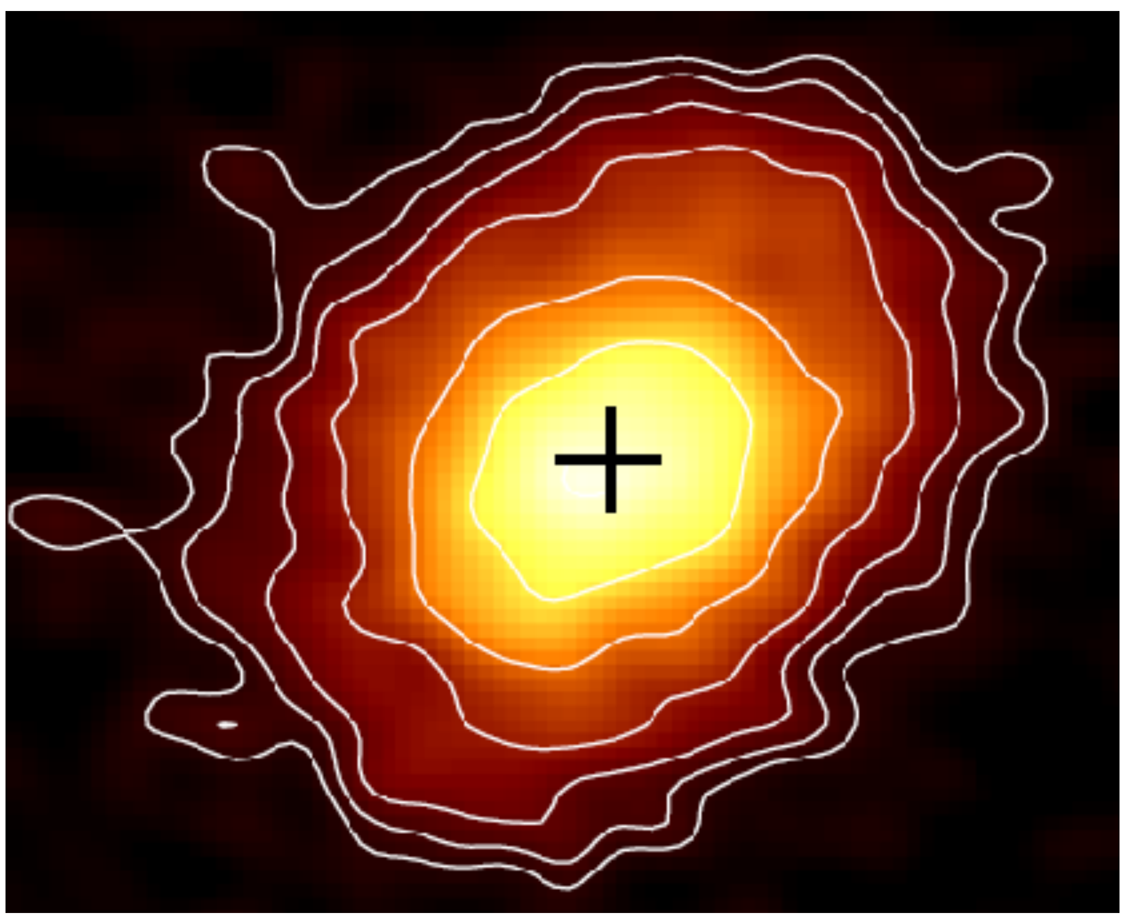
Testi et al. (2014)

Imaging Disks with ALMA Telescope

Atacama Large Millimeter/sub-millimeter Array (2011–)
spatial resolution ~ 0.01 arcsec (視力6000)

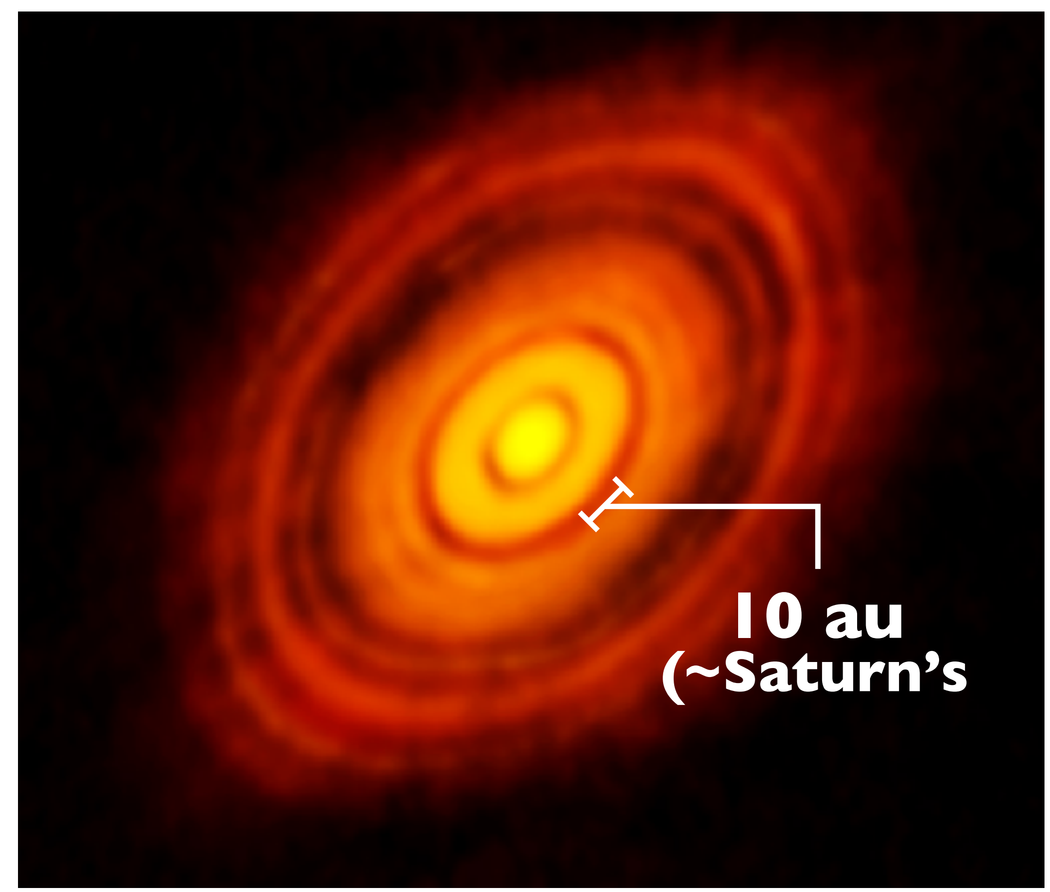
Example: thermal emission from dust around young star HL Tau (140 pc away from us)

CARMA
(resolution ≈ 20 au)



Kwon et al. (2011)

ALMA
(resolution ≈ 4 au)



ALMA Partnership et al. (2015)

Why Do Young Stars Have Rotating Disks?

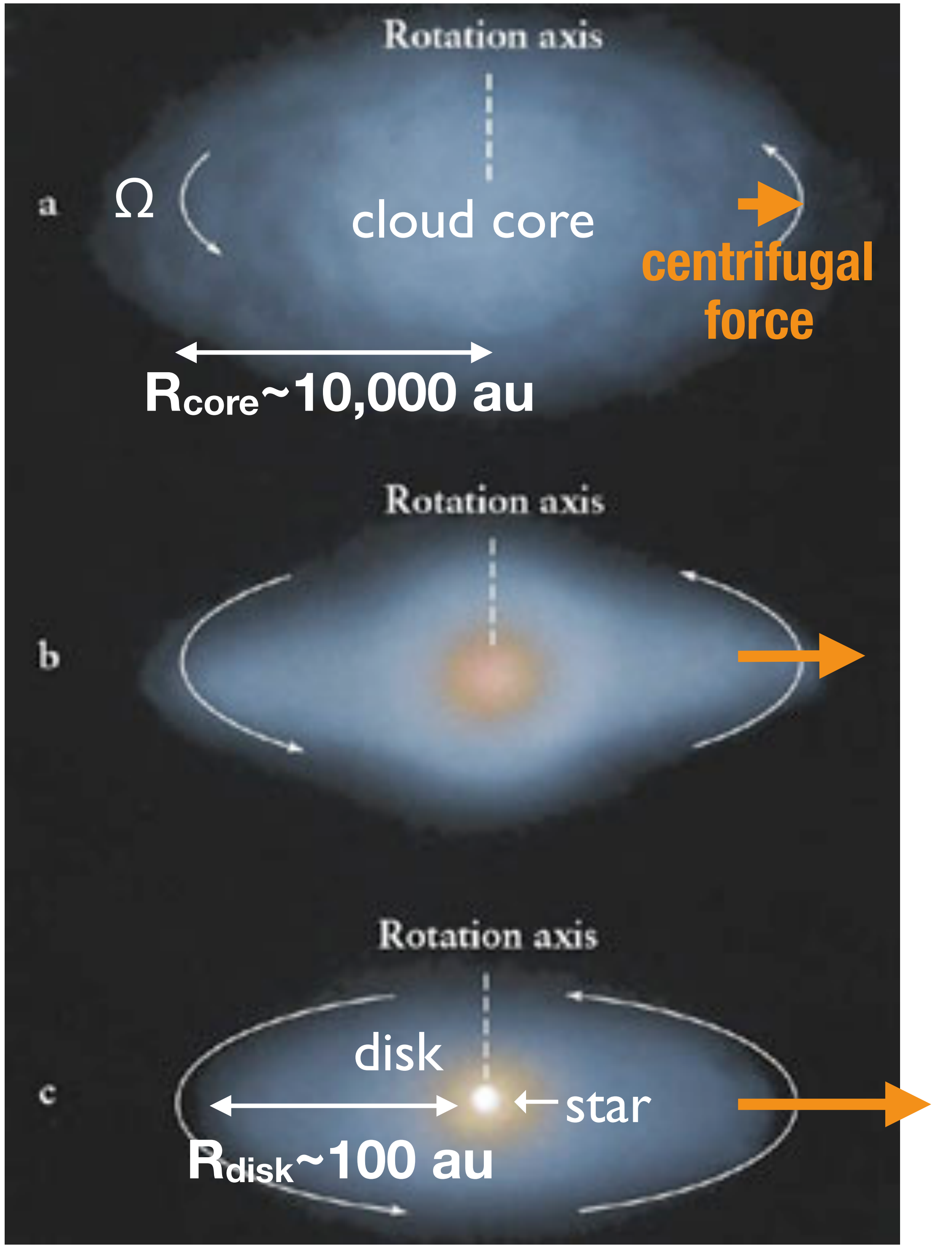
- A star forms from the gravitational collapse of a molecular cloud core (a cold, dense cloud of gas and dust)

- During the collapse, the core's mass M and angular momentum L ~ $MR^2\Omega$ (R: radius, Ω: angular speed) is conserved

➡ The core spins up (Ω increases) as the core shrinks (R decreases)

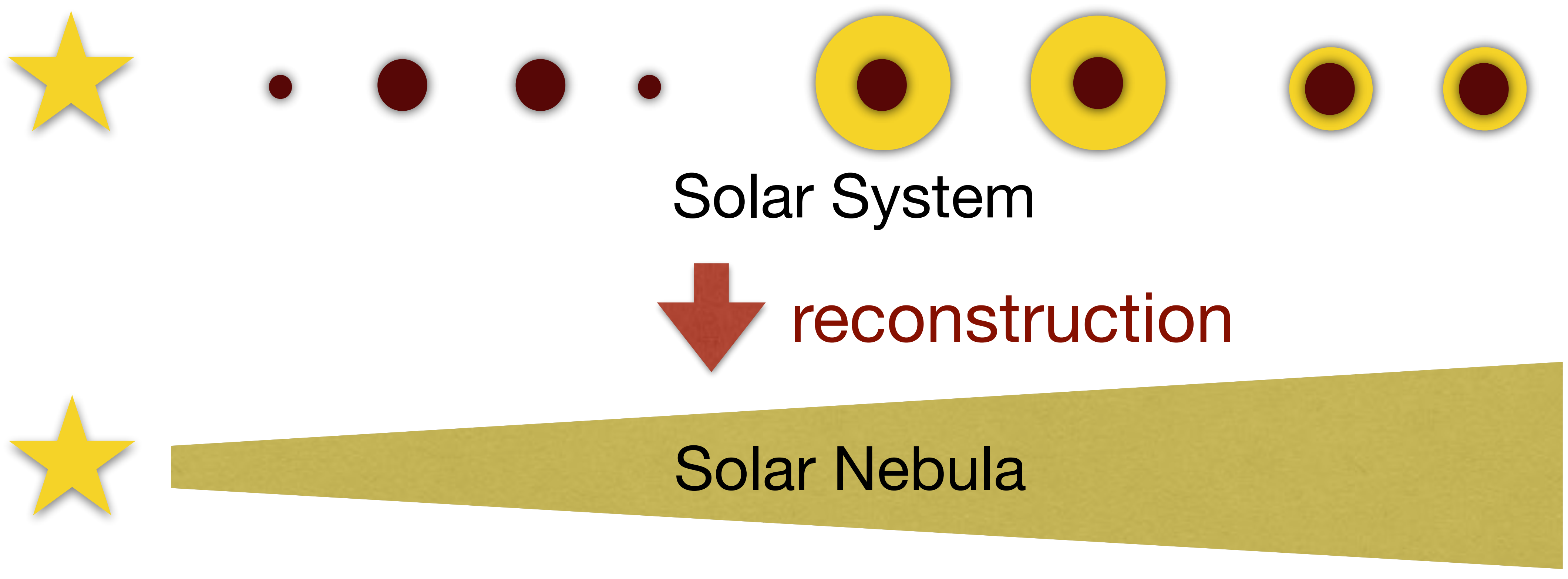
➡ The increased centrifugal force prevents collapse in the direction perpendicular to the rotation axis

➡ A rotating disk forms with a star



The Minimum-mass Solar Nebula Model

(Weidenschilling 1977b; Hayashi 1981)

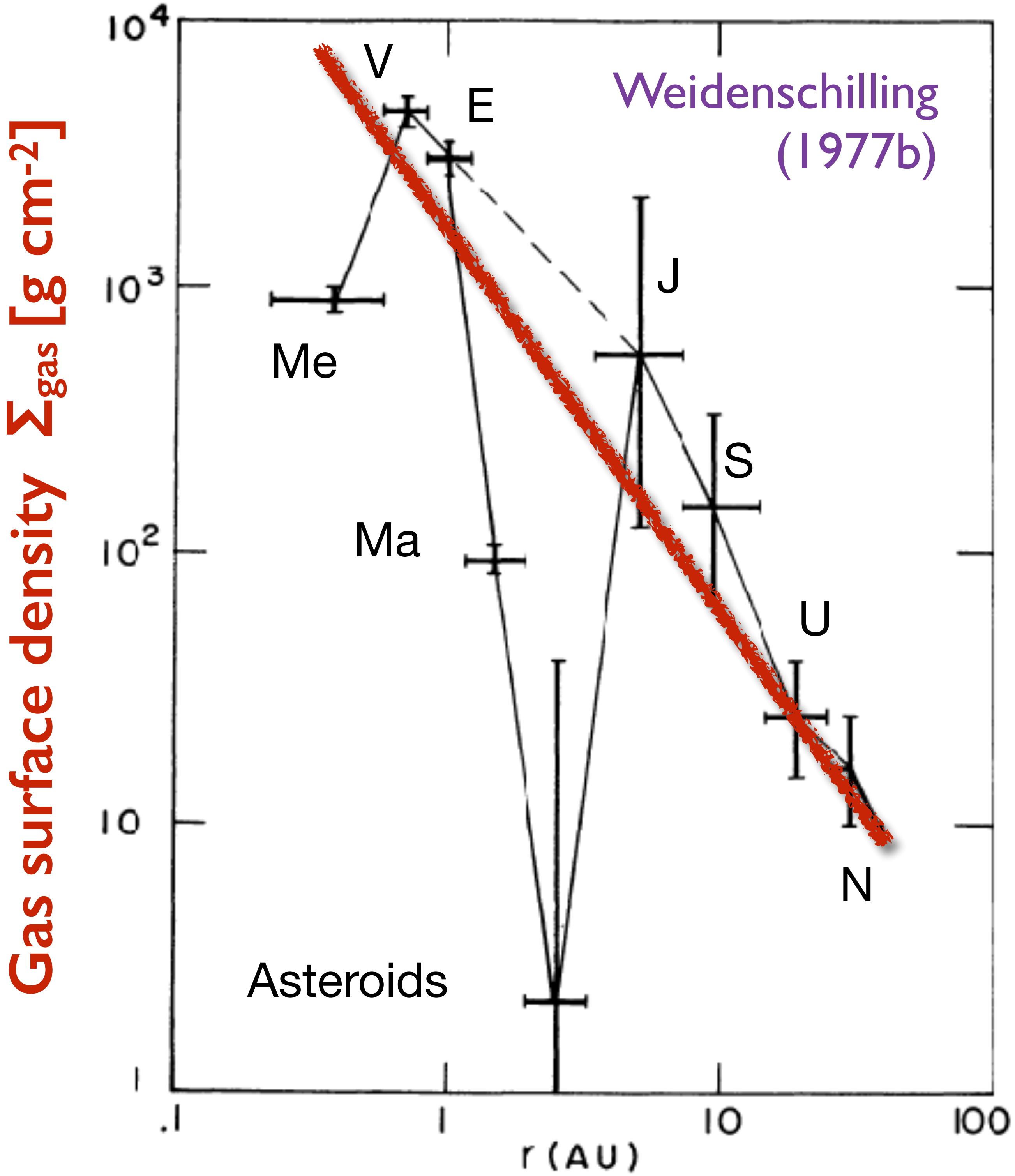


Key Assumptions:

- *All solids* in the disk (nebula) were used to make the planets (planet formation was 100% efficient).
- The planets were born at the present position.
- Gas (H_2, He) mass ~ 100 × dust mass

The Minimum-mass Solar Nebula Model

Surface density Σ: mass per unit area of disk



- The following power-law fit is commonly used:

$$\Sigma_{gas} = 1700\ (r/\mathrm{au})^{-3/2}\ \mathrm{g\ cm^{-2}}$$

$$[\ \Sigma_{dust} \sim 20\ (r/\mathrm{au})^{-3/2}\ \mathrm{g\ cm^{-2}}\ ]$$

- Total disk mass

$$M_{gas} = 0.013\ M_\odot\ (r_{out}/30\mathrm{au})^{1/2}$$

$$[\ M_{dust} \sim 40\ M_\oplus\ (r_{out}/30\mathrm{au})^{1/2}\ ]$$

Dust Mass in Disks from mm Observations

Assuming that the dust disk is optically thin at $\lambda \sim$ mm and that the dust temperature $T_{\rm dust}$ is spatially uniform, the total dust mass $M_{\rm dust}$ in a disk can be estimated as

$$M_{\rm dust} = \frac{F_\nu d^2}{\kappa_\nu B_\nu(T_{\rm dust})}.$$

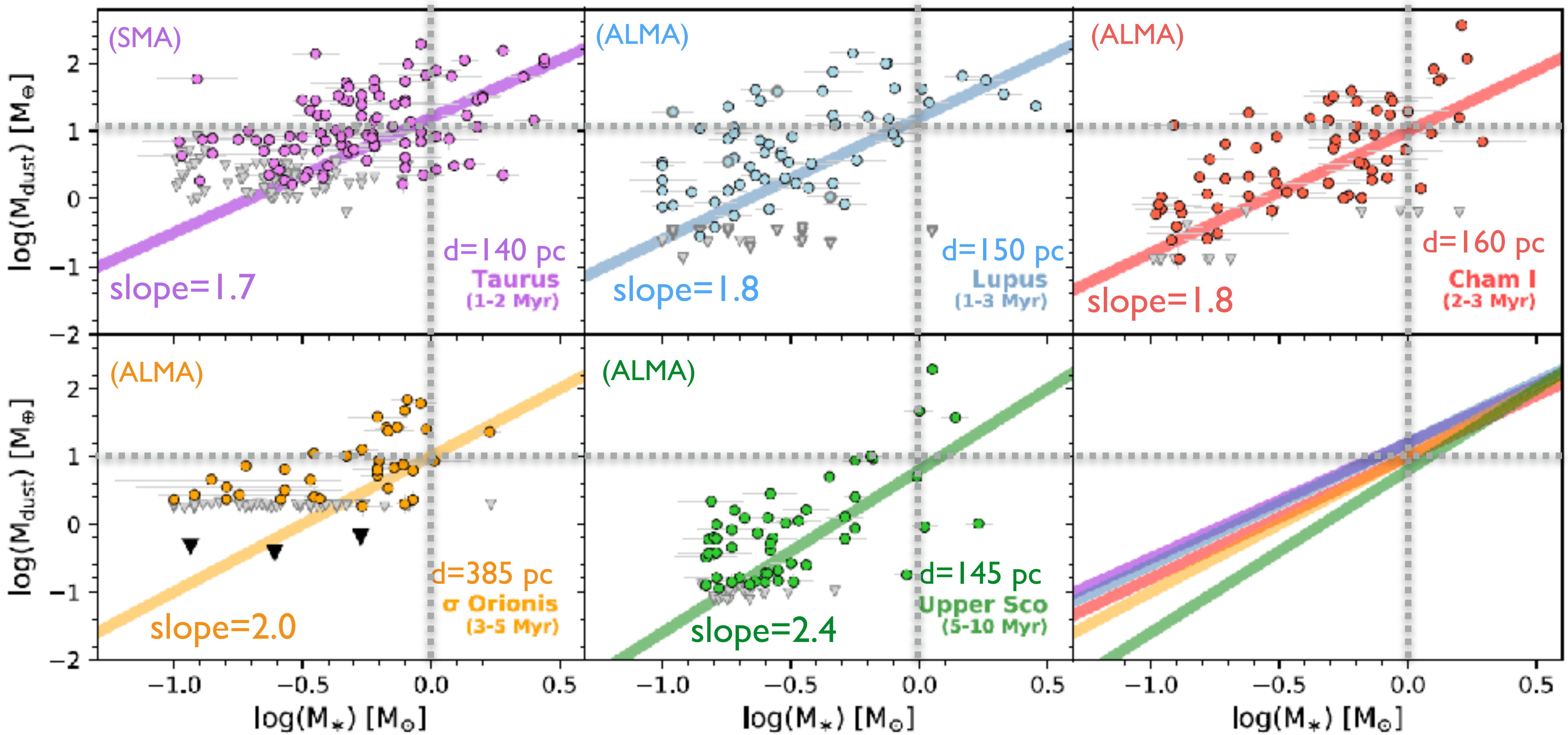
F_ν: flux density (observable)
d: distance to disk
κ_ν: dust opacity (in principle depends on dust size)
B_ν(T): Planck function

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_BT} - 1}$$ (energy flux per unit frequency and unit solid angle)

Dust Mass in Disks from Radio Observations

Dust masses M_{dust} of disks belonging to different star-forming regions

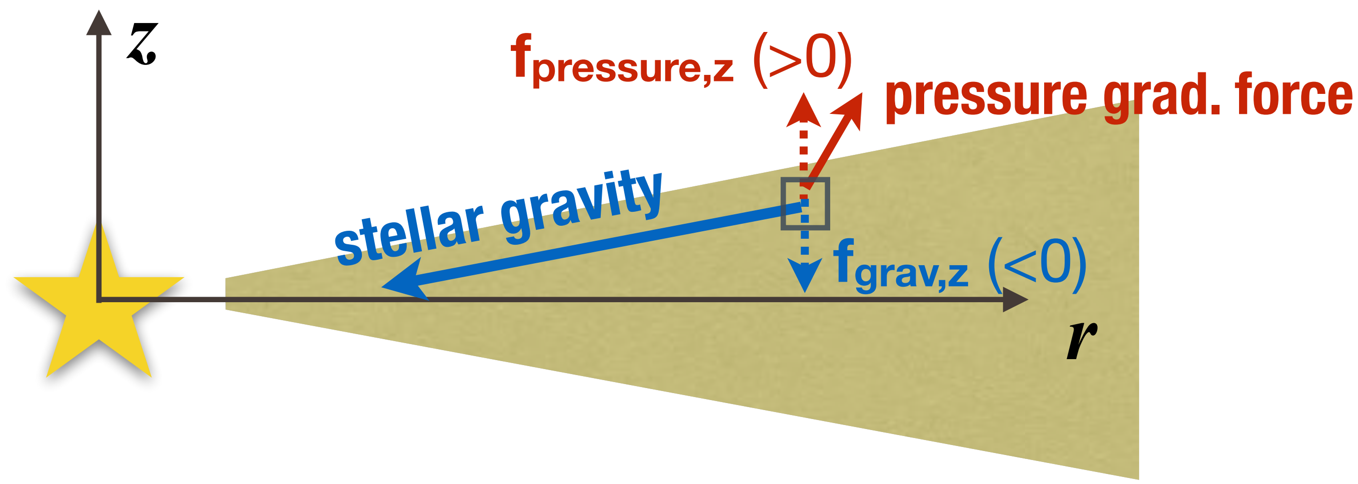
assumption: $\kappa_\nu = 3\ (\lambda/1\mathrm{mm})^{-1}\ \mathrm{cm^2\ g^{-1}}$, $T_{dust} = 20$ K



Ansdell et al. (2017)

For $M_* = M_\odot$, $M_{dust} \sim 10 M_\oplus$

Disk's Vertical Structure



Vertical forces acting on disk gas of unit volume:

- **Stellar gravity**

$$f_{\mathrm{grav},z} = -\rho \frac{GM_* z}{(r^2 + z^2)^{3/2}} \approx -\rho \Omega_{\mathrm{K}}^2 z \quad (r \ll z)$$

$\Omega_{\mathrm{K}} = (GM_*/r^3)^{1/2}$: Keplerian orbital frequency

- **Pressure gradient force**

$$f_{\mathrm{pressure},z} = -\frac{\partial P}{\partial z} = -c_s^2 \frac{\partial \rho}{\partial z}$$

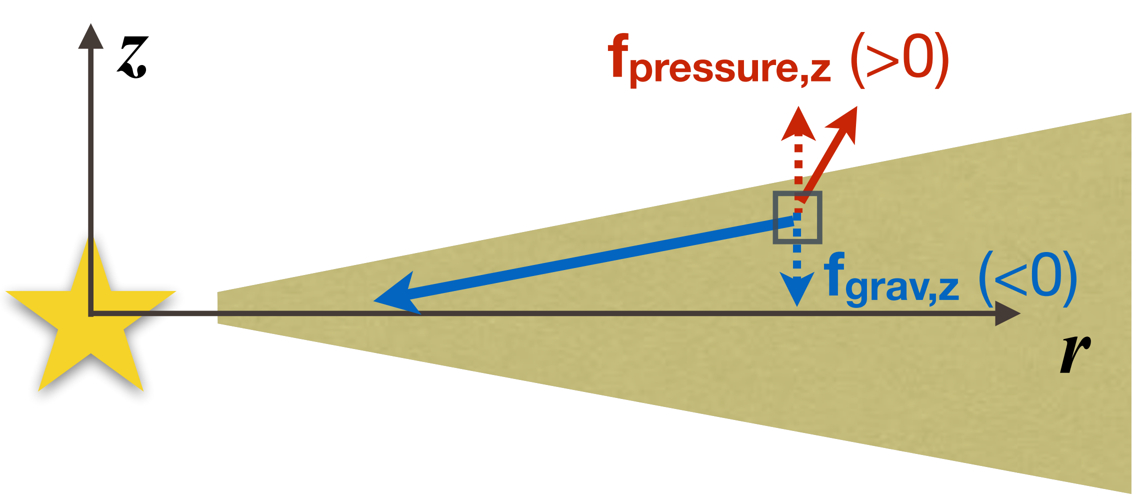
$c_{\mathrm{s}} = (dP/d\rho)^{1/2}$: sound speed

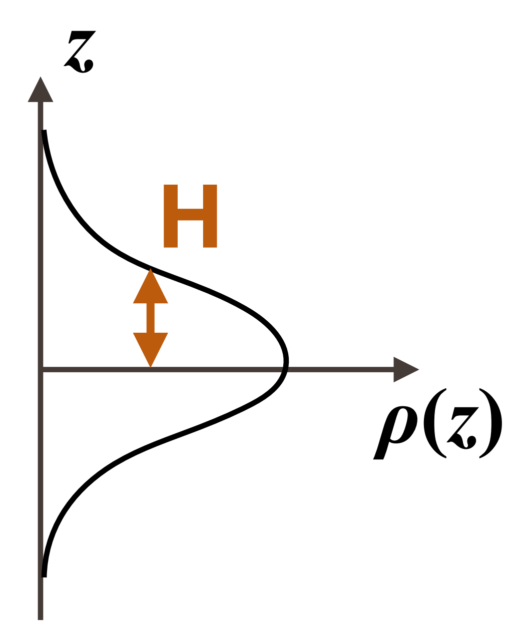
Disk's Vertical Structure

In hydrostatic equilibrium, $f_{\mathrm{grav,z}} + f_{\mathrm{pressure,z}} = 0$,
the vertical profile of gas density becomes Gaussian

$$\rho(z) = \rho_{z=0} \exp\left(-\frac{z^2}{2H^2}\right)$$

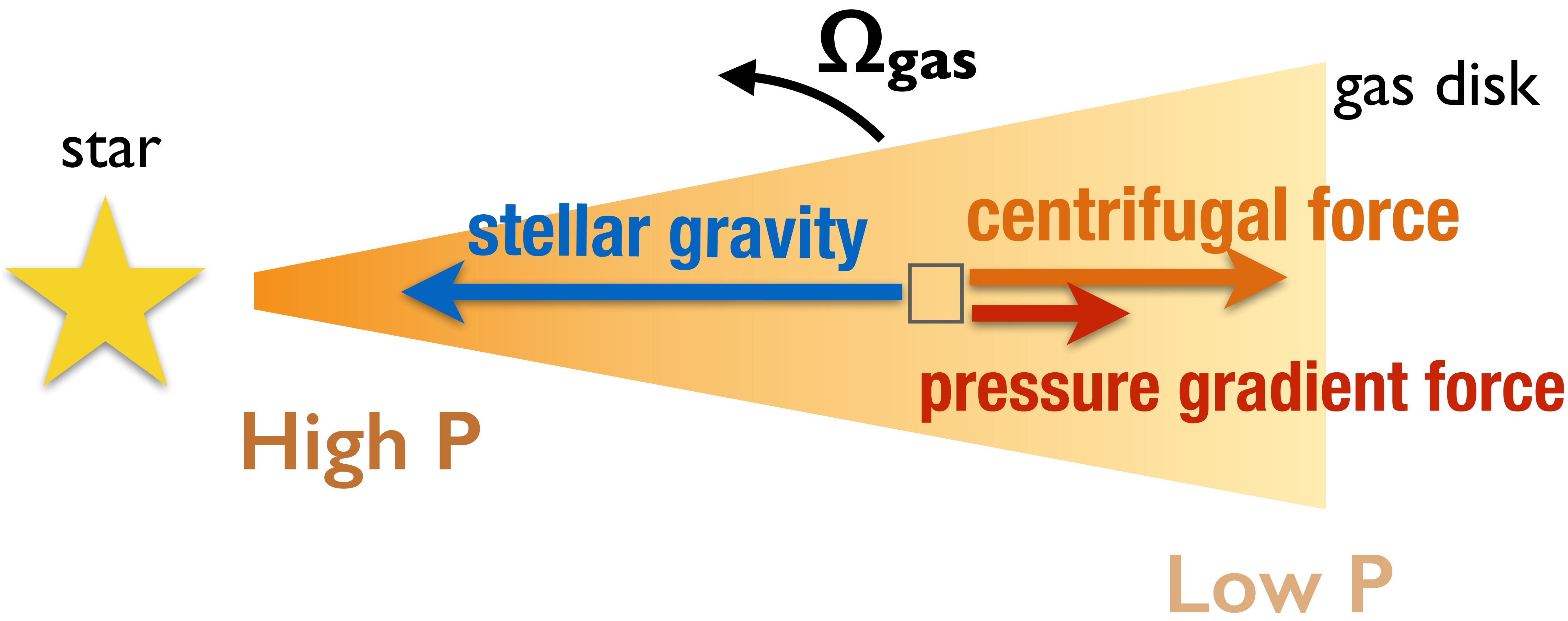
where $H = c_s/\Omega_K$: is called the **scale height** (~disk thickness)





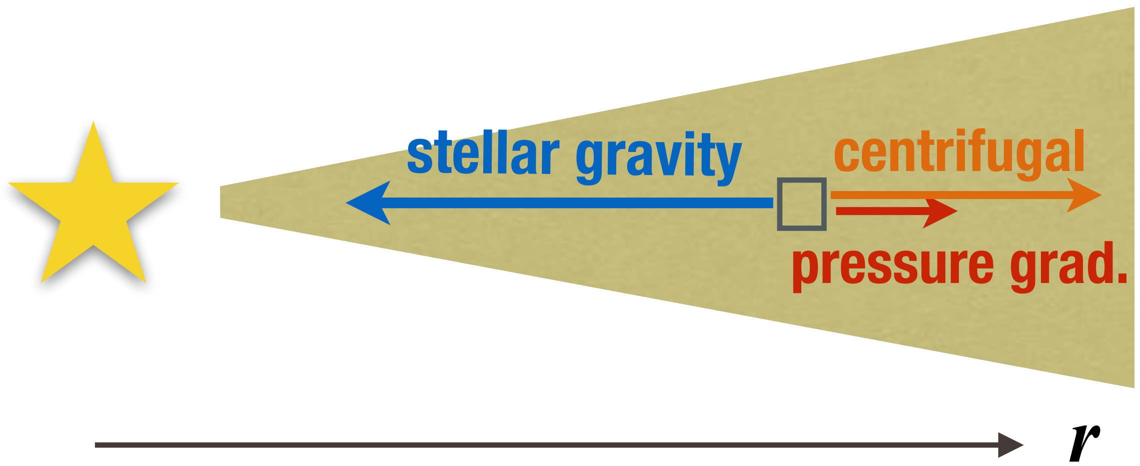
Sub-Keplerian Rotation of the Gas Disk

There are three components of radial forces acting on disk gas:



Because of the presence of pressure gradient, the rotation speed deviates from Keplerian.

Sub-Keplerian Rotation



Per unit volume, the radial forces are:

- **Stellar gravity** $$f_{\mathrm{grav},r} = -\rho \frac{GM_*}{r^2} = -\rho \Omega_{\mathrm{K}}^2 r$$ Ω_{K}: Kepler freq.

- **Pressure gradient force** $$f_{\mathrm{pressure},r} = -\frac{\partial P}{\partial r}$$

- **Centrifugal force** $$f_{\mathrm{cent},r} = +\rho \Omega_{\mathrm{gas}}^2 r$$ Ω_{gas}: orbital frequency **of gas**

Sub-Keplerian Rotation (Cont.)

- $f_{\mathrm{grav},r} + f_{\mathrm{pressure},r} + f_{\mathrm{cent},r} = 0 \Rightarrow$

$$\Omega_{\mathrm{gas}}^2 = \Omega_{\mathrm{K}}^2 + \frac{1}{\rho r}\frac{\partial P}{\partial r}$$

Normally $\partial P/\partial r < 0$ $\Rightarrow$ $\Omega_{\mathrm{gas}} < \Omega_{\mathrm{K}}$ (**sub-Keplerian** rotation)

- Rotational velocity of gas: $u_\phi = r\Omega_{\mathrm{gas}}$

Assuming $|\Omega_{\mathrm{gas}} - \Omega_{\mathrm{K}}| \ll \Omega_{\mathrm{K}}$, we approximately have

$$u_\phi \approx v_{\mathrm{K}} + u'_\phi,$$

$v_{\mathrm{K}} = r\Omega_{\mathrm{K}}$
Kepler velocity

$$u'_\phi = \frac{1}{2}\frac{1}{\Omega_{\mathrm{K}}\rho}\frac{\partial P}{\partial r} = \frac{1}{2}\frac{c_s^2}{v_{\mathrm{K}}}\frac{\partial \ln P}{\partial \ln r}$$

rotation velocity relative to Keplerian

Sub-Keplerian Rotation of the Gas Disk

From radial force balance $f_{\mathrm{grav},r} + f_{\mathrm{pressure},r} + f_{\mathrm{cent},r} = 0,$

$$\Omega_{\mathrm{gas}}^2 = \Omega_{\mathrm{K}}^2 + \frac{1}{\rho r}\frac{\partial P}{\partial r}$$

Normally $\partial P/\partial r < 0$

$\Rightarrow$ $\Omega_{\mathrm{gas}} < \Omega_{\mathrm{K}}$ (sub-Keplerian rotation)

Sub-Keplerian Rotation Speed

Rotational velocity of gas: $\boldsymbol{u_\phi = r\Omega_{\rm gas}}$

Assuming $|\Omega_{\rm gas} - \Omega_{\rm K}| \ll \Omega_{\rm K}$, we approximately have

$$u_\phi \approx v_{\rm K} + \boldsymbol{u'_\phi},$$

where $v_{\rm K} = r\Omega_{\rm K}$ is the Kepler velocity, and

$$u'_\phi = \frac{1}{2}\frac{1}{\Omega_{\rm K}\rho}\frac{\partial P}{\partial r} = \frac{1}{2}\frac{c_s^2}{v_{\rm K}}\frac{\partial \ln P}{\partial \ln r}$$

is the deviation from Keplerian (normally negative)

Magnitude of Sub-Kepler Rotation Speed

$$\eta \equiv -\frac{u'_\phi}{v_K} = -\frac{1}{2}\left(\frac{c_s}{v_K}\right)^2 \frac{d\ln P}{d\ln r} \sim \left(\frac{c_s}{v_K}\right)^2$$

At 1 au around a solar-type star,

- $T \approx 300$ K $\Rightarrow$ $c_s \approx 1$ km/s $\qquad \Rightarrow \ \boldsymbol{\eta \sim 10^{-3}}$
- $v_K \approx 30$ km/s

$$\boldsymbol{u'_\varphi \sim -10^{-3} \times 30 \text{ km/s} \sim -30 \text{ m/s}}$$

This small deviation has little effect on gas disk evolution.

Nonetheless, we will see that this has an *enormous* effect on dust dynamics!